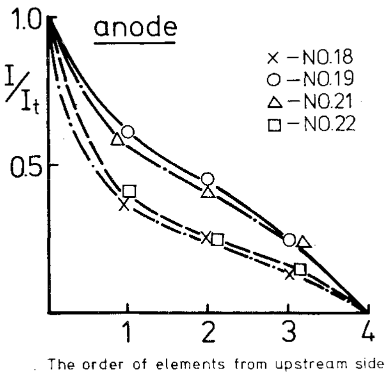

Fig. 3 The current distribution on the anode.

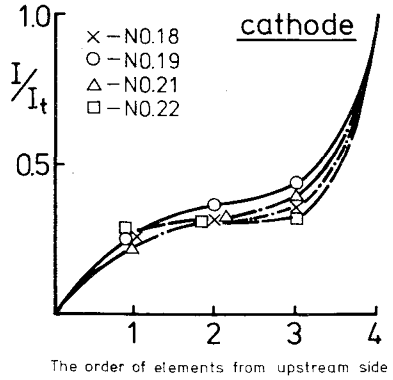

Fig. 4 The current distribution on the cathode.

Figures 3 and 4 show the experimental results of current distributions at the anode and the cathode, respectively. The vertical axis shows the ratio of current (I) accumulated from the downstream edge of the electrode at the anode, or the upstream edge of the electrode at the cathode up to one element to the total current (I_t) (the latter ranged from 12A to 56.6A on one electrode). As is shown in Fig. 3, at the anode, the share of the current of No. 1 element was about 60% for No. 18 and No. 22 electrodes but it reduced to 40% for No. 19 and No. 21 electrodes. This reduced current on No. 1 element was distributed among other elements and therefore, a more uniform current distribution was obtained by this method.

However, the current distribution on the cathode was hardly improved by this method as shown in Fig. 4. These results are caused mainly by the differences in the type of discharge in relation to generating and depositing electrons on each electrode. On the cathode, where electrons are generated, the arc discharges are usually highly concentrated, tend to fix their arc spots on the electrodes, and are affected strongly by the properties of the electrode materials,[4] but it seems that so long as water-cooled metal electrodes are used, the variation of surface temperature of the electrodes, that is, the variation of the interresistances in the boundary-layer regions that contain the liquid and solid seed layers, is not so effective in modifying the current distribution.

On the contrary, on the anode that attracts electrons, the arc discharges are hardly dependent on the properties of electrode materials, and so, in the MHD channel, they are strongly affected by the interelectrode resistances in boundary layers as mentioned in Ref. 5. Although, in this experiment, the surface temperature distributions were determined approximately using assumed values for the thickness of seed layers, the thermal and electrical conductivity of seed material, and other parameters, further study is necessary to obtain the detailed correlations between the current and surface temperature distributions and to make it possible to design in detail this type of electrode for future MHD generators.

Conclusion

A new method is described to control the current distribution on the electrodes in a MHD channel with good results in improving the current distribution on the anode, but with little change for the current distribution on the cathode.

References

[1]Rohatgi, V. K., et al, "Effect of Shape and Resistivity of Electrodes in Faraday MHD Duct," also Yoshikawa K. and Hattori, Y. "Theoretical Investigation of Characteristics of Resistive Electrode with a Load Wire at the Extreme Edge," Petty, S., et al., "Development with the Mark VI Long-duration MHD Generator," 6th International Conference on MHD Electrical Power Generation, Washington, D. C. June 9-13, 1975.

[2]Heywood, J. B. and Womack, H., *Open-cycle MHD Power Generation*, Pergamon Press, London, 1969.

[3]Fushimi, K., et al, "Construction of an MHD Generator with a Large-scale Super-conducting Magnet," 13th Symposium on Engineering Aspects of MHD, Stanford Univ., Stanford, Calif., March 26-28, 1973.

[4]*Handbook for Discharges*, J.I.E.E. of Japan, pp 139-167.

[5]Rubin, E. S. and Eustis, R. H., "Electrode Size Effects in Combustion-driven MHD Generators," 11th Symposium on Engineering Aspects of MHD, Pasaden, Calif., March 24-26, 1970.

On Newtonian Flow Past Power-Law Bodies

Ronald M. Barron*
University of Windsor, Windsor, Ontario, Canada

RECENTLY Hui[1] has developed an iterative method for studying the steady Newtonian flow past two-dimensional (and axisymmetric) bodies of any thickness at zero incidence. To consider the power-law body shape, the discussion is eventually restricted to slender bodies by invoking the hypersonic small-disturbance assumption. The leading terms of Hui's solution for power-law bodies are identical to those of Cole.[2] As Cole points out, these solutions have singularities such as zero density and infinite temperature at the aerofoil surface. These singularities arise because the hypersonic small-disturbance theory does not account for the region of high entropy surrounding the body. As is well-known, the effects of this entropy layer can be treated by the method of matched asymptotic expansions. Using this approach, Ryzhov and Terent'ev[3] have investigated the flow in the entropy layer enveloping bodies supporting power-law shocks, without invoking the Newtonian flow approximation.

In this note a uniformly valid solution under the Newtonian flow approximation is obtained by composition of the "outer" solution of Cole[2] and the "inner" solution of Ryzhov and Terent'ev.[3] However, as will be seen, the existing "inner" solution[3] will have to be improved in order to treat the Newtonian flow problem.

The Newtonian Approximation

Consider a two-dimensional symmetric aerofoil of unit chord length placed in a uniform hypersonic stream of perfect gas at zero incidence. Let the effects of viscosity and heat conduction be negligible. Fix a rectangular coordinate system Oxy so that the origin coincides with the nose of the body, given by $y=y_b(x)$, and the x axis lies along the direction of the freestream. Let u,v,p, and ρ be velocity components in the x and y directions, pressure, and density, respectively.

Following Cole,[2] define

$$y^* = y - y_b(x) \tag{1a}$$

Received May 10, 1976.

Index categories: Shock Waves and Detonations; Supersonic and Hypersonic Flow.

*Assistant Professor, Dept. of Mathematics.

$$v^*(x,y^*) = v(x,y) - u(x,y)y_b(x) \tag{1b}$$

where the prime denotes differentiation with respect to the argument.

The condition that the relative normal component of velocity at the body surface vanishes is then

$$v^*(x,0) = 0 \tag{2}$$

A fundamental small parameter for the Newtonian limit is defined by

$$\epsilon = (\gamma - 1)/(\gamma + 1) \tag{3}$$

where γ is the constant adiabatic exponent of the gas under consideration. Conservation of mass requires that the shock wave be located a distance $0(\epsilon)$ off the surface. However, the shock wave will be assumed to be attached at the nose so that the subsequent theory will not be valid near the leading edge of the aerofoil. To carry out the Newtonian limit $\gamma \to 1$ and $M_\infty \to \infty$ (M_∞ is the freestream Mach number), the coordinate system must be distorted so that distances across the shock layer are measured relative to ϵ.

We seek a solution to the full equations of motion in the form of a power series in ϵ. Thus we write

$$\rho(x,y^*;\gamma,M_\infty) = (1/\epsilon)\rho_n(x,\bar{y};N) + 0(1) \tag{4a}$$

$$u(x,y^*;\gamma,M_\infty) = u_n(x,\bar{y};N) + 0(\epsilon) \tag{4b}$$

$$v^*(x,y^*;\gamma,M_\infty) = \epsilon v_n(x,\bar{y};N) + 0(\epsilon^2) \tag{4c}$$

$$p(x,y^*;\gamma,M_\infty) = p_n(x,\bar{y};N) + 0(\epsilon) \tag{4d}$$

where $\bar{y} = y^*/\epsilon$ and $N = 1/(\epsilon M_\infty{}^2) = \text{constant}$ as $\epsilon \to 0$, $M_\infty \to \infty$.

Substituting expansions (4) into the full equations of motion and retaining only leading terms in ϵ gives the first order Newtonian approximation equations. A further transformation to Von Mises variables via

$$(\partial\psi/\partial x)_{\bar{y}} = -\rho_n v_n \qquad (\partial\psi/\partial\bar{y})_x = \rho_n u_n$$

yields a system of equations equivalent to those of Freeman[4] in his study of Newtonian flow past blunt bodies. Of particular interest for later work is the boundary condition (2) at the body surface, which now takes the form

$$v_n(x,\psi) = 0 \text{ at } \psi = 0 \tag{5}$$

The shock conditions can also be obtained to this approximation by expanding the shock shape $y = y_s(x)$ in the form

$$y_s(x) = y_b(x) + \epsilon y_{sn}(x) + 0(\epsilon^2) \tag{6}$$

Since the shock is to be attached at the nose,

$$y_{sn}(0) = 0 \tag{7}$$

Under the Newtonian approximation, the entropy in the shock layer is given by p_n/ρ_n and must be constant on streamlines, that is $p_n/\rho_n = f(\psi)$. On the shock, $\psi = y_s(x)$, and the appropriate shock conditions allow f to be represented parametrically by

$$f(x) = y'^2(\sigma)/\{1 + y'^2(\sigma)\}, \quad \sigma = y_s^{-1}(\psi) \tag{8}$$

where y_s^{-1} is the inverse function of y_s. Restricting our discussion to power-law shocks $y_s(x) = x^m$, Eqs. (8) become

$$p_n/\rho_n = m^2\{m^2 + \psi^{2(1-m)/m}\}^{-1} \tag{9}$$

Equation (9) is valid across the entire shock layer away from the nose of the aerofoil. Further, as indicated by Cole,[2] we take $N = 0$ for power-law bodies.

Outer Solution

Cole[2] has obtained the outer solution by making the additional assumption that the body is slender. Changing his notation for our convenience, his results for a power-law shock wave are

$$p_n^{(0)}(x,\psi) = mx^{-2(1-m)}\{2m - 1 + (1-m)\psi/x^m\} + 0(x^{-4(1-m)}) \tag{10a}$$

$$\rho_n^{(0)}(x,\psi) = \{(2m-1)/m + (1-m)\psi/(x^m m)\}(\psi/x^m)^{2(1-m)/m} + 0(x^{-2(1-m)}) \tag{10b}$$

$$u_n^{(0)}(x,\psi) = 1 - (m^2/2)\{\psi^{-2(1-m)/m} + x^{-2(1-m)}\} + 0(x^{-4(1-m)}) \tag{10c}$$

$$\begin{aligned} v_n^{(0)}(x,\psi) = {} & \{2m^2(1-m)/(2m-1)(3m-2)\}x^{-(1-m)} \\ & \times(\psi/x^m)^{(3m-2)/m} \times F(2,(3m-2)/m,\ [(3m-2)/m] \\ & + 1;\ -(1-m)(\psi/x^m)/(2m-1)) + \{m^2(1 \\ & - m)(2-m)/2(2m-1)^3\}x^{-(1-m)}(\psi/x^m)^{2(2m-1)/m} \\ & \times F(2, 2(2m-1)/m,\ [2(2m-1)/m] + 1;\ -(1 \\ & - m)(\psi/x^m)/(2m-1)) + 0(x^{-3(1-m)}) \end{aligned} \tag{10d}$$

where $2/3 < m < 1$ for finite drag, the superscript (0) denotes "outer" solution and $F(a,b,c;z)$ is the hypergeometric function. The solutions (10) are valid near the shock, i.e., for large ψ, but not in the entropy layer enveloping the aerofoil surface.

Inner Solutions

Ryzhov and Terent'ev[3] have studied the flow in the entropy layer for aerofoils supporting power-law shock waves, with $2/3 < m < (2\gamma+2)/(3\gamma+2)$. Their solution will now be considered in the Newtonian limit as $\epsilon \to 0$ and $M_\infty \to \infty$, $(2/3 < m < 4/5)$. Altering their notation, we find

$$p_n^{(i)}(x,\psi) = m(2m-1)x^{-2(1-m)} + 0(x^{-4(1-m)}) \tag{11a}$$

$$u_n^{(i)}(x,\psi) = 1 - (1/2)m^2\{m^2 + \psi^{2(1-m)/m}\}^{-1} + 0(x^{-4(1-m)}) \tag{11b}$$

where the superscript (i) denotes "inner" solution.

To calculate $v_n^{(i)}$ we recast the Ryzhov and Terent'ev solution in the appropriate form using Eq. (1b). Hence consider

$$v^{*(i)} = v^{(i)} - y_b'(x)u^{(i)} \tag{12}$$

where (see Ref. 3)

$$v^{(i)}(x,\psi) = mx^{-(1-m)}\lambda_0\{1 - 2\gamma(\gamma+1)^{-2}h_0^{(\gamma-1)/\gamma}m^2[m^2 + \psi^{2(1-m)/m}]^{-1/\gamma}x^{-2(1-m)(\gamma-1)/\gamma}\} + \ldots \tag{13a}$$

$$u^{(i)}(x,\psi) = 1 - 2\gamma(\gamma+1)^{-2}h_0^{(\gamma-1)/\gamma}m^2[m^2 + \psi^{2(1-m)/m}]^{-1/\gamma}x^{-2(1-m)(\gamma-1)/\gamma} + \ldots \tag{13b}$$

$$y_b(x) = \{\lambda_0 + R(0)x^{-m+2(1-m)/\gamma}\}x^m + \ldots \tag{13c}$$

and

$$R(\psi) = -\epsilon h_0^{-1/\gamma} m^{-2/\gamma}\{m^{m/(1-m)} N_0 - \psi$$
$$\times F(1/\gamma, m/[2(1-m)],\ [m/(2-2m)]+1;$$
$$-m^{-2}\psi^{2(1-m)/m})\} \tag{13d}$$

Here λ_0 is the surface value of Sedov's[5] self-similar variable λ, h_0 is the ratio of the pressure on the surface to that immediately behind the shock, and

$$N_0 = \Gamma[(2-m)/(2-2m)]\Gamma[-m/(2-2m)+1/\gamma]/\Gamma(1/\gamma)$$

Using Eq. (13) in (12) yields

$$v^{*(i)}(x,\psi) = \epsilon 2(1-m)\gamma^{-1} h_0^{-1/\gamma} m^{-(2/\gamma)+m/(1-m)}$$
$$\times N_0 x^{-1+2(1-m)/\gamma} + \ldots \tag{14}$$

which is $O(\epsilon)$ as required by Eq. (4). However, $v^{*(i)}(x,0) \neq 0$ so that the surface boundary condition (5) is not satisfied. A careful investigation of the equations and results of Ref. 3 shows that the next term in the expansion of $v^{(i)}$ is of the order $x^{-1+2(1-m)/\gamma}$ and hence contributes to the leading term in $v^{*(i)}$. In fact $v^{(i)}$ should be expanded as

$$v^{(i)}(x,\psi) = 2(\gamma+1)^{-1} m x^{-(1-m)}\{V_1(\psi)$$
$$+x^{-2(1-m)(\gamma-1)/\gamma} V_2(\psi) + x^{-m+2(1-m)/\gamma} V_3(\psi)\} + \ldots$$

where

$$V_1(\psi) = (1/2)(\gamma+1)\lambda_0 \tag{15a}$$

$$V_2(\psi) = -\gamma(\gamma+1)^{-1}\lambda_0 h_0^{(\gamma-1)/\gamma} m^2 [m^2 + \psi^{2(1-m)/m}]^{-1/\gamma} \tag{15b}$$

$$V_3(\psi) = [(1-m)(\gamma+1)/(m\gamma)] R_2(\psi) \tag{15c}$$

Using Eq. (15) in place of Eq. (13a) in Eq. (12) yields the corrected Newtonian approximation

$$v_n^{(i)}(x,\psi) = [2(1-m)/(2m^2-m)] x^{-(2m-1)}\psi$$
$$\times F(1, m/(2-2m), [m/(2-2m)]+1;\ -m^{-2}\psi^{2(1-m)/m})$$
$$+ \ldots \tag{11c}$$

We note that for a given power-law shock $y = x^m$, ($2/3 < m < 4/5$), the body shape is obtained from Eqs. (13c) and (13d). From Brocher[6] we extract the Newtonian expansion of λ_0 as

$$\lambda_0 = 1 - \epsilon\Lambda + 0(\epsilon^2)$$

where

$$\Lambda = [m/(2m-1)]^{2(1-m)/m}\int_{(2m-1)/(1-m)}^{\infty} \sigma^{-(3m-2)/m}$$
$$\times (1+\sigma)^{-1} d\sigma$$

Thus, to order ϵ

$$y_b(x) = x^m - \epsilon\{\Lambda x^m$$
$$+ m^{(2m-1)/(1-m)}(2m-1)^{-1} N_1 x^{2(1-m)}\} + \ldots \tag{16}$$

so that Eq. (6) gives

$$y_{sn}(x) = \Lambda x^m + m^{(2m-1)/(1-m)}(2m-1)^{-1} N_1 x^{2(1-m)}$$
$$+ \ldots; \qquad N_1 \equiv \lim_{\gamma\to 1} N_0 \tag{17}$$

From Eq. (16), one sees that, for the asymptotic theory employed here, the body and the shock coincide up to a station $x = x_0$, where

$$x_0 \equiv \{-N_1(2m-1)^{-1}\Lambda^{-1} m^{(2m-1)/(1-m)}\}^{1/(3m-2)} \tag{18}$$

Composite Solution

Newtonian approximate solutions, which are uniformly valid for large x and across the entire shock layer can be obtained by matching the outer solutions (10) and the inner solutions (11). These composite solutions are

$$p_n(x,\psi) = m x^{-2(1-m)}\{2m-1+(1-m)\psi/x^m\} + \ldots \tag{19a}$$

$$\rho_n(x,\psi) = x^{-2(1-m)}\{(2m-1)/m + (1-m)\psi/(m x^m)\}$$
$$\times\{m^2 + \psi^{2(1-m)/m}\} + \ldots \tag{19b}$$

$$u_n(x,\psi) = 1 - (m^2/2)\{m^2+\psi^{2(1-m)/m}\}^{-1}$$
$$-(m^2/2)x^{-2(1-m)} + \ldots \tag{19c}$$

$$v_n(x,\psi) = 2m^2(1-m)(2m-1)^{-1}(3m-2)^{-1}x^{-(1-m)}$$
$$\times(\psi/x^m)^{(3m-2)/m}F[2,(3m-2)/m,[(3m-2)/m]+1;$$
$$-(1-m)(2m-1)^{-1}\psi/x^m] + (m^2/2)(1-m)(2-m)$$
$$\times(2m-1)^{-3}x^{-(1-m)}(\psi/x^m)^{2(2m-1)/m}F[2,$$
$$2(2m-1)/m,[2(2m-1)/m]+1;-(1-m)$$
$$\times(2m-1)^{-1}\psi/x^m[+2(1-m)m^{-1}(2m-1)^{-1}x^{-(2m-1)}$$
$$\times\psi F[1,m/(2-2m),[m/(2-2m)]+1;-m^{-2}\psi^{2(1-m)/m})$$
$$-2m^2(1-m)(2m-1)^{-1}(3m-2)^{-1}x^{-(2m-1)}$$
$$\times\psi^{(3m-2)/m} + \ldots \tag{19d}$$

We note that Eq. (9) has been used to derive ρ_n and that $2/3 < m < 4/5$. Solution (19) represents the asymptotic expansions, valid for large distances downstream, of the Newtonian approximation to the solution of the full steady flow problem.

Acknowledgment

This work is based in part on the author's Ph.D. dissertation at Carleton University, Ottawa, Canada. The author wishes to express his gratitude to P. Mandl for his invaluable advice and discussion.

References

[1] Hui, W. H., "On Axisymmetric and Two-Dimensional Flow with Attached Shock Waves," *Astronautica Acta*, Vol. 18, Feb. 1973, pp. 35-44.

[2] Cole, J. D., "Newtonian Flow Theory for Slender Bodies," *Journal of Aerospace Sciences*, Vol. 24, June 1957, pp. 448-455.

[3] Ryzhov, O. S., and Terent'ev, E. D., "On the Entropy Layer in Hypersonic Flows with Shock Waves Whose Shape is Described by a Power Function," *Journal of Applied Mathematics and Mechanics*, Vol. 34, Jan. 1970, pp. 461-477.

[4] Freeman, N. C., "On the Theory of Hypersonic Flow Past Plane and Axially Symmetric Bluff Bodies," *Journal of Fluid Mechanics*, Vol. 1, Oct. 1956, pp. 366-387.

[5] Sedov, L. I., *Similarity and Dimensional Methods in Mechanics*, Academic Press, New York, 1959.

[6] Brocher, E. F., "On Similar Solutions for Strong Blast Waves and Their Application to Steady Hypersonic Flow," *Journal of Aerospace Sciences*, Vol. 29, June 1962, pp. 694-701.